Chemistry 240 Semester 01-2009
Homework for Submission #2

Answer the following question and submit them for marking on or before Thursday 5th February in class. If any answers show evidence of copying, the whole exercise will attract zero marks. Please note that presentation of your answer (not neatness as such!) is extremely important.

1) Radon is a radioactive noble gas. It is produced in granitic rocks by the radioactive decay of uranium. It has a short half-life, but apart from that would not be expected to present a significant threat at altitude. Demonstrate the truth of this statement by applying the barometric formula to show that if the gas is expelled at sea level the number of radon atoms at 100 m is expected to be considerably diminished. Take the temperature as 27°C. (Hint: calculate N_1/N_0.)

The Barometric formula may be stated as

$$\frac{p_1}{p_0} = e^{-Mg(h_1-h_0)/RT}$$

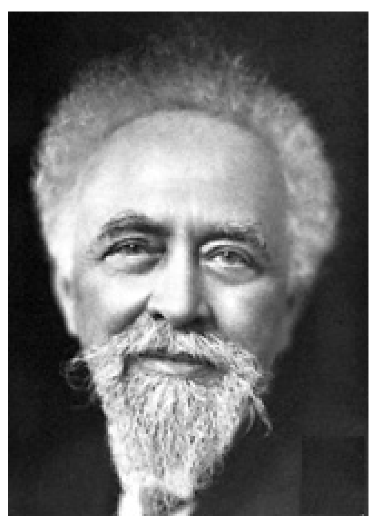
where P_1 is the pressure at height h_1 and P_0 is the pressure at height h_0, M is the molar mass of the gas in kg mol^{-1}, g the acceleration due to gravity and T the temperature.

But since PV=nRT and n = N/L, where N is the number of molecules and L is Avogadro's number,

$$\frac{P_1}{P_0} = \frac{N_1}{N_0} = e^{-Mg(h_1-h_0)/RT} = e^{-222\times10^{-3}\times9.81\times(100-0)/\{8.31\times(273+27)\}} = \underline{\underline{0.916}} \text{ to 3 sig. figs.}$$

(This compares with a figure of 0.989 at 100 m for nitrogen, showing that the concentration of radon drops much more rapidly than that of a lighter gas.)

2) Jean Perrin, a Frenchman (shown on the right), was awarded the Noble prize in 1922. In 1909 he conducted a beautiful series of experiments on sedimentation equilibria. These are governed by the Boltzmann distribution as shown below:



$$\frac{N_1}{N_0} = e^{-mg(h_1-h_0)/kt}$$

He examined a suspension of fine gamboge particles of radius 0.212 μm in water. These have a density of 1.207×10^{-3} kg m^{3} as compared with that of water of 1.00×10^{3} kg m^{-3}. This gives each of the particles an effective mass of $\frac{4}{3}\pi r^3(\rho_g - \rho_w)$ where ρ_g represents the density of the gamboge and ρ_w the density of the water.

Derive the expression

$$\ln(N) = \frac{-4\pi r^3(\rho_g - \rho_w)gh}{3kT} + \ln(N_0)$$

for the relative numbers of gamboge particles at height h (N) as compared with the number at height $h_0 = 0$ (N_0) from the Boltzmann distribution.

Answer

$$\frac{N_1}{N_0} = e^{-mg(h_1 - h_0)/kt}$$

But $m = \frac{4}{3}\pi r^3(\rho_g - \rho_w)$, N_1 becomes N, h_1 becomes h and $h_0 = 0$

$$\therefore \frac{N}{N_0} = e^{-\frac{4}{3}\pi r^3(\rho_g - \rho_w)g(h-0)/kt} = e^{\left(\frac{-4\pi r^3(\rho_g - \rho_w)gh}{3kt}\right)}$$

$\therefore$ Taking natural logarithms of both sides, we have:

$$\ln\left(\frac{\mathrm{N}}{\mathrm{N}_0}\right) = \left(\frac{-4\pi r^3(\rho_g - \rho_w)gh}{3kt}\right)$$

$$\therefore \ln N - \ln N_0 = \left(\frac{-4\pi r^3(\rho_g - \rho_w)gh}{3kt}\right)$$

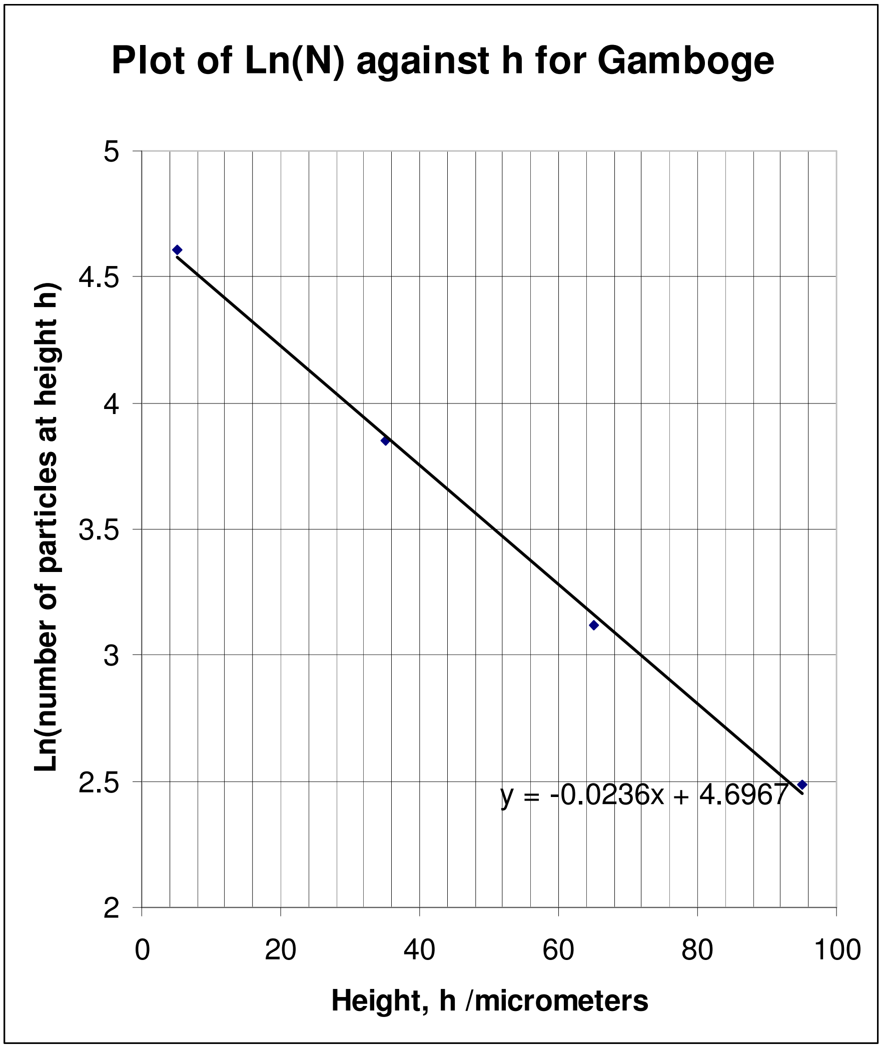
$$\therefore \ln N = \left(\frac{-4\pi r^3(\rho_g - \rho_w)gh}{3kt}\right) + \ln N_0$$

The following data are taken from one of Perrin's experiments:

Height h / μm	5	35	65	95
N (relative units)	100	47	22.6	12

Plot ln(N) against h and from the slope of the straight-line graph determine k. Given that R = 8.31 $JK^{-1}mol^{-1}$, g = 9.81 ms^{-2} and T = 20°C, calculate L (Avogadro's number). (This was an early experiment, and only gives a rather rough estimate of L.)

Answer



Slope of graph is -0.0236$(\mu\text{m})^{-1}$ = -0.0236$\times(10^{-6}\ \text{m})^{-1}$ = -0.0236$\times 10^{6}\ \text{m}^{-1}$.

$$\therefore -0.0236\times 10^{6} = \frac{-4\pi r^{3}(\rho_g - \rho_w)g}{3kT}$$

$$\therefore k = \frac{4\pi r^{3}(\rho_g - \rho_w)g}{3\times 0.0236\times 10^{6}T} = \frac{4\times 3.142\times(0.212\times 10^{-6})^{3}\times(1.207-1.00)\times 10^{3}\times 9.81}{3\times 0.0236\times 10^{6}\times(20+273)} = 1.172\times 10^{-23}$$

$$\therefore L = \frac{R}{k} = \frac{8.31}{1.172\times 10^{-23}} = \underline{\underline{7.089\times 10^{23}}}\text{mol}^{-1}$$

(Of course, we now know that the value of L is 6.02…$\times 10^{23}$, but at that time the value above was a reasonable estimate.)